1. Classic Measure

The volatility measure corresponds to the standard deviation. It is the square root of the average of each variation to the average variation. The formula is:

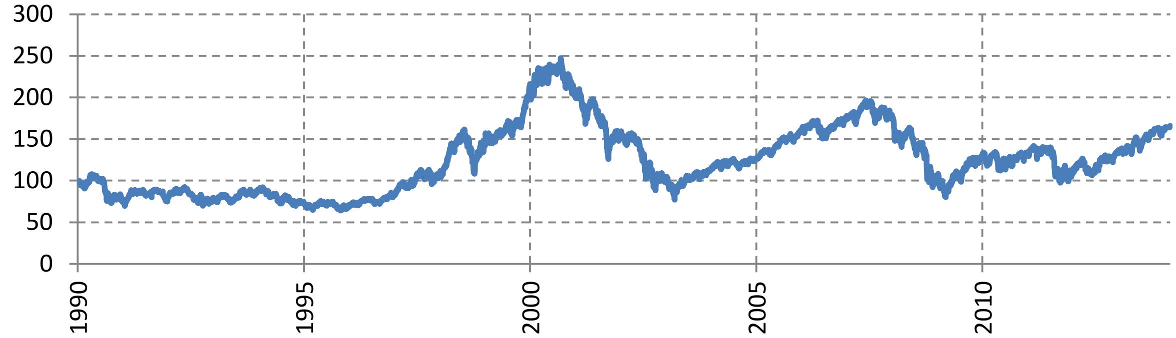
$$\sigma_X = \sqrt{E[(X - E[X])^2]} = \sqrt{E[X^2] - E[X]^2} \text{ with } E[X] = \frac{1}{n}\sum_{i=1}^{n} X_i$$

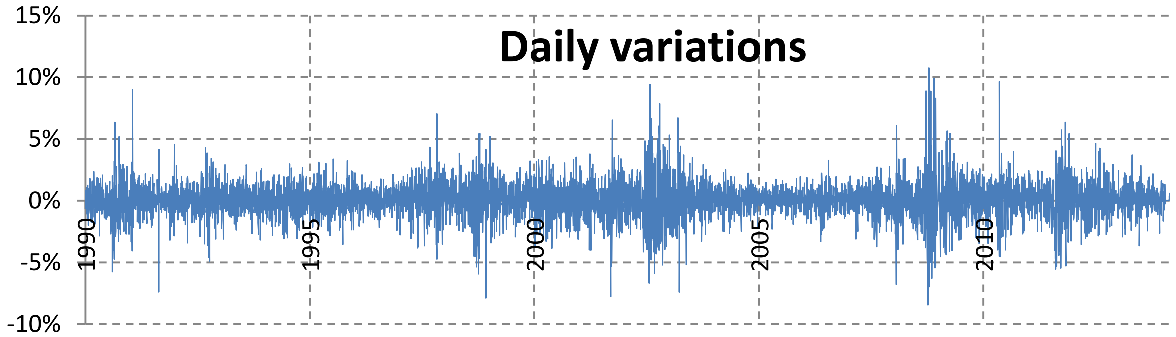
The standard calculation is made on 256 daily variations (~1 year) and is annualized (multiply by $\sqrt{256} = 16$).

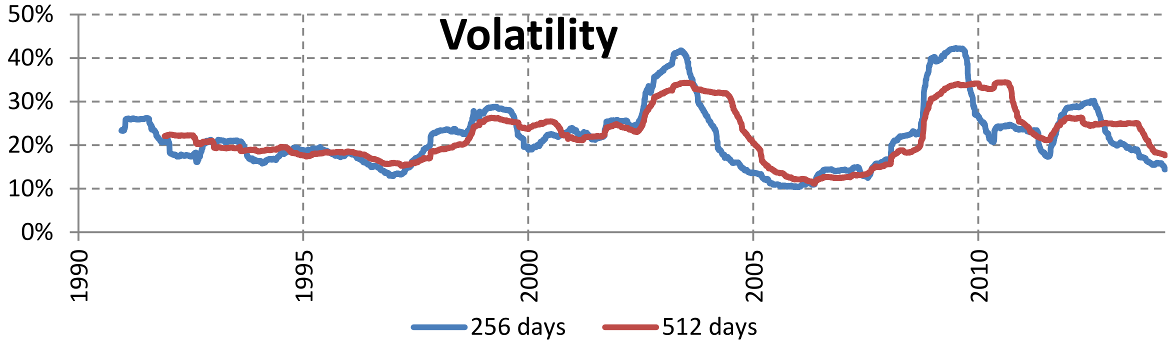
2. Problematic

- Measure stability
- Length of historic sensitivity
- Recent variations low sensitivity

Example on CAC 40 Future Spot:







As we can see on the graphs page 1, the volatility measure is very sensitive to the length of historic. This sensitivity can lead to take this risk measure into account either too much or too low, depending on the length of historic which the fund manager or the risk controller decided to use.

3. Reflections and solutions

As length of historic leads to instability of this measure, the first improvement will be to overweight recent variations and to decrease the weight of past variations. Thanks to this, if someone decides to use 512 days instead of 256 days, the last 256 days will have a low weight and so will have a low impact on the measure. Moreover, with more weight on recent variations, the new measure will be more reactive and will reflect better the current market conditions.

We now have to define the profile of the weighting. We could define a weight for the most recent variation and a weight for the oldest variation, but this could lead to attribute a different weight to a same variation, depending on the length of historic, which would not be logical. We prefer to use an exponential measure with an amortizing factor which would allow having a minimum of 90% for the sum of the weights for the target length of historic.

To calculate this exponential volatility (facto f), we use the same formula than for classic volatility, but with

$$E[X] = \frac{\sum_{i=1}^{n} w_i * X_i}{\sum_{i=1}^{n} w_i} \text{ and } w_i = (1-f) * f^{i-1}$$

This is a sum of geometric suit, so we can deduce:

$$\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} (1-f) * f^{i-1} = 1 - f^n$$

So:

$$E[X] = \frac{1-f}{1-f^n} * \sum_{i=1}^{n} f^{i-1} * X_i$$

$$E[X^2] = \frac{1-f}{1-f^n} * \sum_{i=1}^{n} f^{i-1} * X_i^2$$

Moreover, as we want that the sum of the weight correspond to a minimum of 90%, the minimum length of historic will be:

$$1 - f^{length\ of\ historic} > 90\%$$

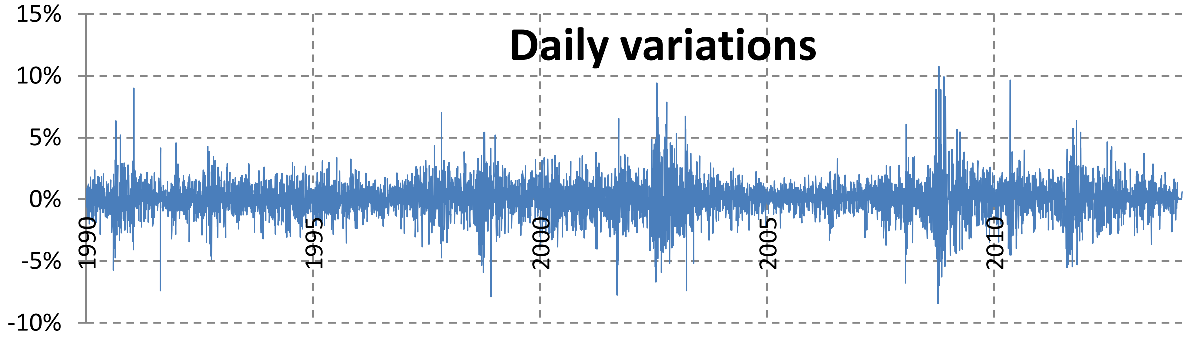
So:

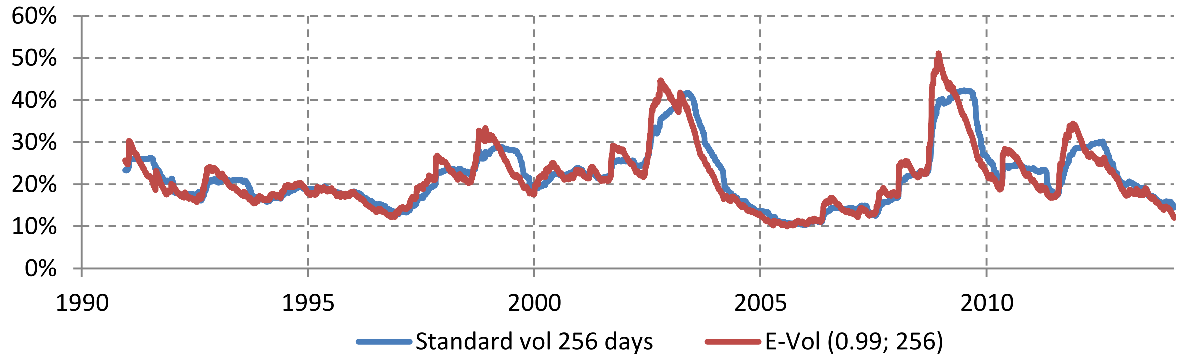
$$length\ of\ historic > \frac{\ln(1 - 90\%)}{\ln(f)}$$

4. Results

1. Quality of the measure

We will test this exponential volatility with a factor of 0.99. The minimum length of historic is 230 days, so with 256 days it will be ok. We compare the standard volatility 256 days with the exponential volatility over the same period.

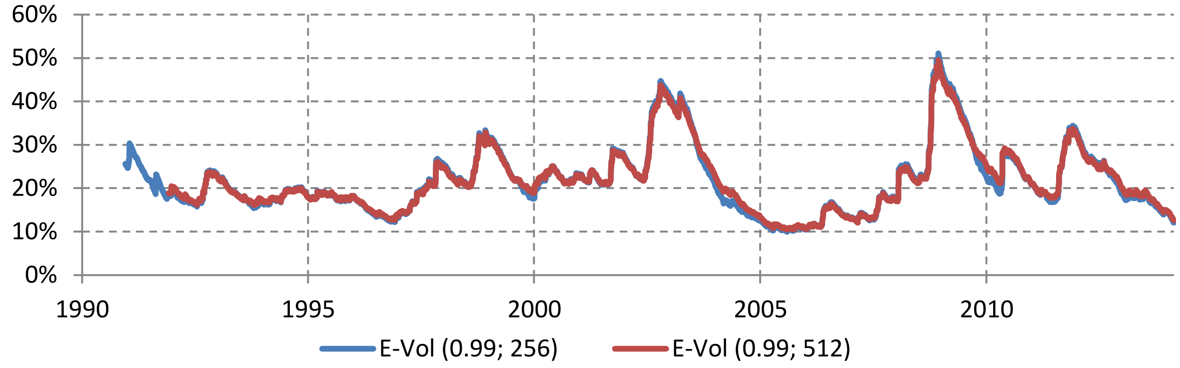


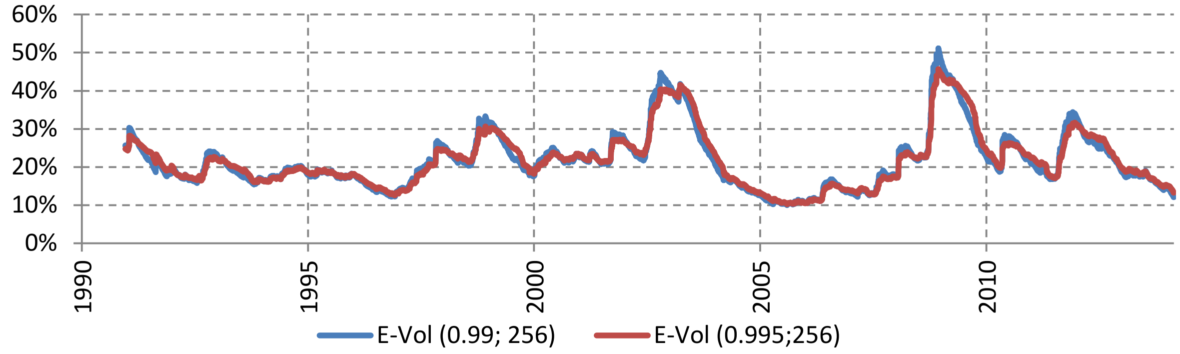


As one can see, the exponential volatility is more reactive to market behavior. This measure is a better proxy to recent market risk.

2. Stability of the measure

We will test 2 length of historic: 256 and 512 days. We will then compare the E-Vol (0.99; 256) to E-Vol (0.995; 256) to test the stability of the measure, even in a case when the sum of the weights is below 90%.





As we can see, this new measure is very stable. Length of historic is now not a problem. Moreover, this new measure seems to adapt very quickly to recent market conditions and so reflect very well the risk on the market.